O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5713 Linear Systems
Fall 2003
Midterm Exam #2

PH.D. Students- DO ALL FIVE PROBLEMS

MS Students- Choose FOUR out of five problems: Indicate below which four are chosen

1).____ 2).____ 3).____ 4).____

Name : ______________________________

Student ID: ________________________________

E-Mail Address:_______________________________________

Problem 1:

Let $F = \{0,1,2,3,4\}$. Define the rules of addition and multiplication such that F is a field.

Problem 2:
For

$$A = \begin{bmatrix} 3 & 2 & 1 & 0 \\ 3 & 2 & 1 & 0 \\ 3 & 2 & 1 & 0 \end{bmatrix}$$

Determine the rank and nullity of the above linear operator, A ? And find a basis for the range space and the null space of the linear operator, A, respectively ?

Problem 3:

Assume $\alpha = [a_1 \quad a_2 \quad \cdots \quad a_n]^T$ and $\beta = [b_1 \quad b_2 \quad \cdots \quad b_n]^T$ are linearly independent, prove that $\alpha' = [a_1 \quad a_2 \quad \cdots \quad a_n \quad a_{n+1}]^T$ and $\beta' = [b_1 \quad b_2 \quad \cdots \quad b_n \quad b_{n+1}]^T$ are linearly independent as well.

Problem 4:

Consider the subspace of $\Re^4$ consisting of all 4×1 column vector $x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$ with constraints $x_1 + x_2 + x_3 = 0$ and $3x_1 + 3x_2 + 3x_3 = 0$. Extend the following set (with only one element) to form a basis for THE subspace:

$$\begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \end{bmatrix}.$$

<u>Problem 5</u>:

Let A be an $m \times n$ matrix. Show that the set of all $1 \times m$ vectors y satisfying $yA = 0$ forms a vector space, called the left null space of A, with dimension $m - \rho(A)$, where $\rho(A)$ is the rank of A.